Korean J. of Chem. Eng., **14**(4), 257-262 (1997)

ANALYSIS AND OPERATION OF COOLING WATER FLOWS IN A HEAT EXCHANGERS NETWORK

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(Received 26 February 1997 • accepted 29 May 1997)

Abstract – In general, the flow rates of cooling water flowing in a heat exchangers network should be measured for its operational purpose. However, it is difficult to install flow meters on every spot because of the financial and technical reasons. This work presents an explicit algorithm for determining the flow rates of cooling water flowing in the heat exchangers network in the chemical plant. An algorithm is also presented to obtain the desired flow rates of cooling water in each heat exchanger. In order to solve this problem, multivariable optimization techniques will be used to minimize the differences between the desired flow rates and the present cooling water flow rates which flows through the heat exchanger. The commercial computer program GAMS/MINOS which solves the non-linear optimization problem is used. Numerical examples are presented to illustrate the scope of this work which can be handled with the formulation.

Key words: *Heat Exchangers Network, Cooling Water Flows Analysis, Pipe Network, Newton-Raphson, GAMS, Nonlinear Optimization*

INTRODUCTION

Steady-state analysis of flows and pressures in a pipe networks system is a problem of great importance in engineering [Nielsen, 1989; Ormsbee and Wood, 1986]. Various mathematical models have been evolved for the analysis of water distribution systems [Demuren and Ideriah, 1986]. An iterative solution of the full pipe network system provides a powerful technique for the determination of steady-state pressures and flows conditions in the pipe network [Boulos and Wood, 1990]. Today most water distribution systems are designed and analyzed by using computer algorithm [Chansler and Rowe, 1990]. There are two basic principles involved in analyzing networks: one is the algebraic sum of flows at all nodes is zero and the other is the algebraic sum of the head loss of all pipes around any closed loop must be zero. Several mathematical algorithms have been developed for computer analysis of water distribution systems [Hammer and Mark, 1986; Walski et al., 1990; Male and Walski, 1990]. The cooling water supplied for many heat exchangers in chemical plants is supplied and retained in a system. That is, a cooling water network system consists of the cooling water supplying pump, the pipes, and cooling towers, etc. Fig. 1 shows the simple cooling water system in a heat exchangers network.

In general, the flow rates of cooling water flowing in a heat exchangers network should be measured for its operational purpose. However, it is difficult to install the flow meters on every spot for measuring the flow rates of cooling water. Since the diameters of pipes in a network are mostly bigger than 10 inches and the number of heat exchangers is more than 100, economic or technical problems should be encountered. To solve this problem, this work presents an explicit algorithm for determining the flow rates of cooling water flowing in the heat exchangers network. In terms of numerical analysis, there are two general approaches to solving networks problems: one is where the heads at each node are the unknowns and the other is where the flows are the unknowns [Tullis, 1989]. We will use the first systematic method for developing algorithm. The problem solution is based on the steady-state equations which describe the cooling water which flows in the heat exchangers network. Since the equations are non-linear, Gauss-Siedel method or Newton-Raph-

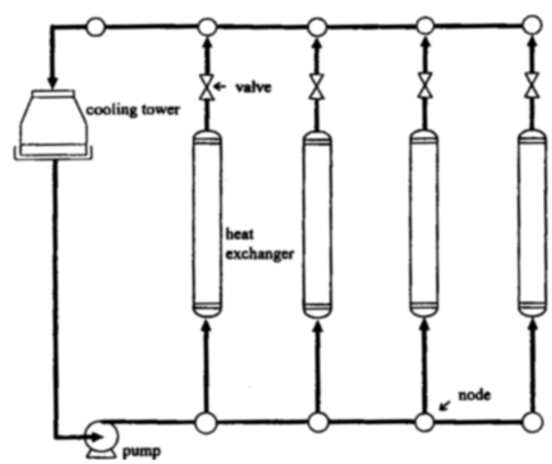

Fig. 1. A cooling water system of a heat exchangers network.

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son method will be applied to solve the system of equations.

Another problem arises in the existing heat exchangers network. It is the manipulation problem of the flow rates of cooling waters in order to obtain the desired flow rates in each heat exchanger. Sometimes, even if engineers operate valves located at the rear of heat exchangers extremely, they can not obtain the desired flow rates of cooling water. In this case, diameters of pipe located at the rear of heat exchanger should be changed. Since all heat exchangers are connected to a cooling water system, the operation of the valve located at the rear of a heat exchanger should affect the flows of all the other heat exchangers. Therefore, the overall manipulations of the flows in all heat exchangers should be considered simultaneously. In this work, an operation problem of cooling water flows passing through the heat exchanger will be studied. In order to solve this problem, the multivariable optimization techniques will be applied to minimize the differences between the desired cooling water flows and the present cooling water flows passing through the heat exchangers. The technology to obtain the optimal solutions to the large highly nonlinear mathematical programming problems has just become available in the recent past by the introduction of such models as GRG2, MINOS, and GAMS/MINOS [Mays and Tung, 1992; Brooke et al., 1992]. In this study, we use the commercial program GAMS/MINOS which can solve the nonlinear programming problem effectively.

ANALYSIS OF COOLING WATER FLOWS IN A NETWORK

1. The Choice of Experimental Equations

The Hazen-Williams equation, the Darcy-Weisbach equation and the Manning equation are useful to calculate flow rates and head losses of water in pipes. Engineers in the waterworks field generally use the Hazen-Williams equation [Chansler and Rowe, 1990]. For water distribution systems it, gives reasonably good results when compared with the more theoretically correct the Darcy-Weisbach equation. Also it avoids the need to update the friction factors with each iteration, as would have to be done when using the Darcy-Weisbach equation [Walski et al., 1990]. The Hanzen-Williams equation can be expressed in the form as follows [Hammer and Mark, 1986; Tullis, 1989].

$$Q = C_H D^{2.63} \left(\frac{\Delta H}{C \cdot L} \right)^{0.54} \quad (1)$$

Where Q is the flow rates between two nodes, C_H is Hazen-Williams roughness coefficient, D is pipe diameter, L is pipe length, C is conversion factor and ΔH is head losses, respectively.

2. The Derivation of Simultaneous Equations for Network

The distribution of flows through a heat exchangers network under a certain loading pattern must satisfy the conversation of mass. Each node is defined that they are connections of two or more pipes or where flows are removed from or supplied into the system. The flows of cooling water at each node must be conserved, that is, mass balance at node

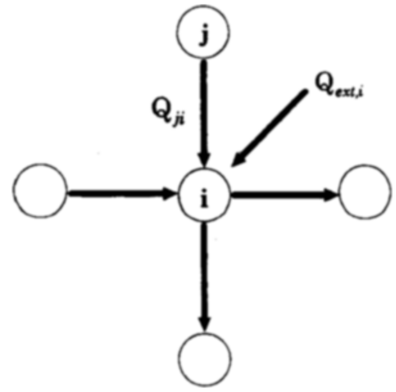

Fig. 2. Node connections with the external flow.

i (i=1,···,n) is as follows,

$$\sum_{j=1}^{n} Q_{ji} + Q_{ext,i} = 0 \qquad \text{for node i (i=1,}\cdots\text{,n)} \quad (2)$$

where Q_{ji} is the flows from the node j to the node i. $Q_{ext,i}$ is the external flows at the node i. That is, when flows remove from or supplied into the system, they have negative or positive sign. Therefore, Eq. (2) expresses the mass balance of water flows at the node i (i=1,···,n). Fig. 2 shows the flows between any central node i and any node j connected node i.

Then Hanzen-Williams equation is substituted into mass balance equations to calculate unknown cooling water flows with head as variable at each node. To apply, Hazen-Williams equation is rearranged by constant part, that is,

$$Q_{ji} = A_{ji}(H_j - H_i)^{0.54} \quad \text{where, } A_{ji} = \frac{C_H D_{ji}^{2.63}}{(C \cdot L_{ji})^{0.54}} \quad (3)$$

Eq. (3) is substituted for Q_{ji} which is in the mass balance equations Eq. (2) and new equation Eq. (4) is obtained.

$$\sum_{j=1}^{n} A_{ji}(H_j - H_j)^{0.54} + Q_{ext,i} = 0 \quad (4)$$

Eq. (4) is rearranged with respect to the head at each node, that is, nonlinear equations with the same number of unknowns which is the total number of nodal heads are obtained as follows.

$$H_i = \frac{\sum_{j=1}^{n} B_{ji} H_j + Q_{ext,i}}{\sum_{j=1}^{n} B_{ji}} \quad \text{where, } B_{ji} = A_{ji} \frac{1}{|H_j - H_i|^{0.46}} \quad (5)$$

The nodes connected to the cooling water supplying pumps have the fixed external flow rates in the heat exchangers network. For the purpose of solving the problem, the external flows, $Q_{ext,i}$, at these nodes are being treated as positive constants in the formulation. On the contrary, cooling waters passed through the heat exchangers network are returned to cooling towers and small amounts of cooling waters are drained periodically from some heat exchangers for the blowdown purpose. At these nodes, external flows, $Q_{ext,i}$, are being treated as variables which have negative signs in the formulation.

The other nodes, at which no external flow exists, have $Q_{ext,i}$ =0 as constant values. In order to solve the Eqs. (4), the total sum of external flows, $\sum_i Q_{ext,i}$ should have the value of zero. In addition, heads, H_i, are treated as variables at the nodes which have constant values of $Q_{ext,i}$, in Eq. (4). Also, heads are treated as constants at the nodes which have the unknown $Q_{ext,i}$. The constant values of heads, H_i, are given with the elevation (m) of the nodes in the formulation.

When applied at all the nodes, Eq. (5) yields a system of nonlinear simultaneous equations in the unknown heads and external flows. The nonlinear simultaneous equations could be solved in an iterative solution technique such as Gauss-Siedel method or Newton-Raphson method [Chapra and Canale, 1988].

3. Network Simulation Algorithm

Gauss-Siedel method and Newton-Raphson method could be applied to solve the heat exchangers network problem. The algorithm of searching for solutions is described in Fig. 3. In the algorithm, the total sum of external cooling water flows at each node, $|\sum_i Q_{ext,i}|$, converges on error boundary, until iteration is stopped. A numerical example will be presented in the results and discussions section.

3-1. The Use of Gauss-Siedel Method

The nonlinear simultaneous equations expressed as Eq. (5) can be solved by iteration from node 1 to node n. In due course, heads of every node converge on ultimate values, but when it converges it often do so very slowly [Chapra and Canale, 1988].

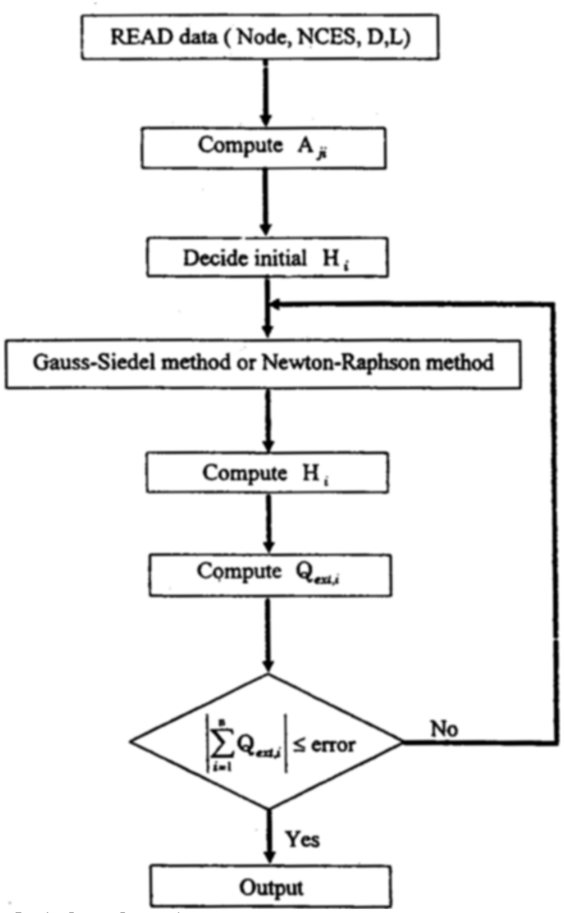

Fig. 3. A flowchart for network analysis algorithm.

3-2. The Use of Newton-Raphson Method

Gauss-Seidel method is convenient for use, but it takes too much time for solving a large network. Therefore, we will apply the Newton-Raphson method which can solve the nonlinear simultaneous equations effectively. To apply the Newton-Raphson method, the simultaneous equations Eq. (4) are reconsidered as follows.

$$F_i(H_j,\cdots,H_n) = \sum_{j=1}^{n} A_{ji}(H_j - H_i)^{0.54} + Q_{ext,i} = 0 \quad (i=1,\cdots,n) \tag{6}$$

Then the Newton-Raphson method can be applied to the Eq. (6). That is,

$$\begin{bmatrix} \frac{\partial F_1}{\partial H_1} & \cdots & \frac{\partial F_1}{\partial H_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_n}{\partial H_1} & \cdots & \frac{\partial F_n}{\partial H_n} \end{bmatrix} \cdot \begin{bmatrix} \Delta H_1 \\ \vdots \\ \Delta H_n \end{bmatrix} = \begin{bmatrix} F_1 \\ \vdots \\ F_n \end{bmatrix} \tag{7}$$

Where $\Delta H = H_i^{k+1} - H_i^k$ and H_i^k is H_i's value which is obtained after the Newton-Raphson method is iterated by k times. Eq. (7) is solved by elimination method such as Gauss-Jordan method.

MANIPULATION OF COOLING WATER FLOWS

In the introduction, the differences between the flow rates of the present cooling water passing through the heat exchangers and the desired flow rates of the cooling water were mentioned. If this case occurred, the heat exchangers could not be operated properly. In this case, operating engineers can adjust valves located at the rear of heat exchangers in order to manipulate flow rates of cooling waters passing through the heat exchangers.

However, since all heat exchangers are connected to a cooling water system, overall manipulations of the flows in all heat exchangers should be considered at the same time. In this study, we will deal with the adjustment of valve openings using multivariable optimization techniques.

1. Equivalent Lengths of Valves

As a matter of fact, equivalent length of a pipe which includes a valve, L_e consists of real pipe length and valve equivalent length.

$$L_e = L_{real} + L_v \tag{8}$$

The relations of valve equivalent lengths and the degrees of valve openings can be found in many references [Ludwig, 1977; Mays and Tung, 1992; Male and Walski, 1990; Walski et al., 1990]. Through the polynomial curve fittings, the relationships between valve equivalent lengths and valve openings in the references can be expressed as Eq. (9). For simplicity, curve fittings are made in third order ploynomials.

$$L_v = a\delta^3 + b\delta^2 + c\delta + d \tag{9}$$

The data of constants a, b, c, d for the gate valves are described in Fig. 4 for various diameters. Where L_v is valve equivalent length and δ is the degrees of valve openings.

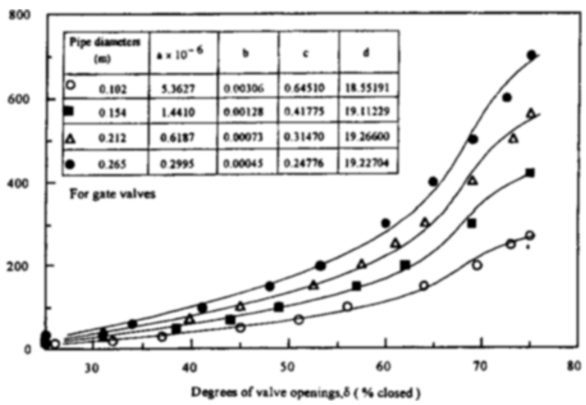

Fig. 4. The relationships between valve equivalent lengths and degrees of valve openings for gate valves.

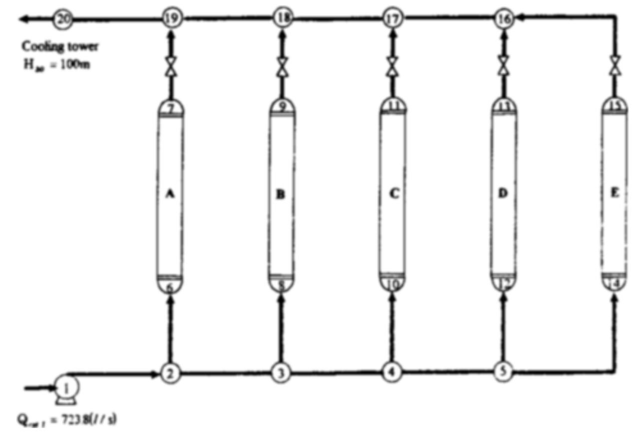

Fig. 5. A heat exchangers network for Example 1.

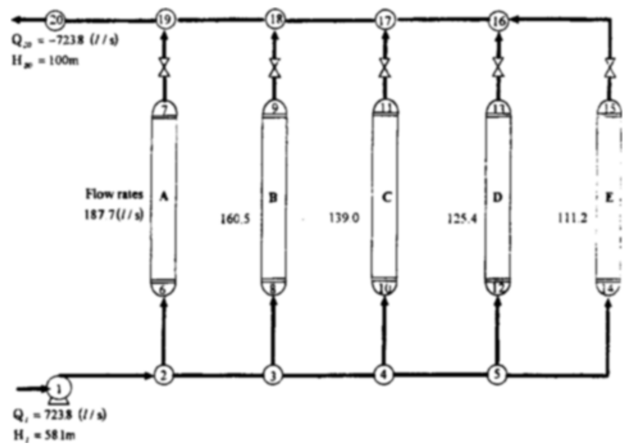

Fig. 6. Calculated flow rates between nodes for Example 1.

2. Optimization Problem

The manipulation problem of cooling water flows in a heat exchangers network can be defined as optimization problem as follows.

$$\text{Minimize} \sum_{ji\in S} |Q_{ji} - Q_{ji}^D| \qquad \text{for (j,i)} \in S \tag{10}$$

Subject to Eq. (4)

Decision variable $L_{e,ji}$ for (j,i) ∈ P

Q_{ji} and Q_{ji}^D indicate the flow rates and the desired flow rates of each heat exchangers. The set S and P denote node connections of heat exchangers and node connections of pipe located at the rear of heat exchangers, respectively. In this work, minimization of calculated differences between the present flow rates and the desired flow rates is obtained due to GAMS/MINOS which is the nonlinear programming optimization solver. GAMS is a programming language that is used for the mathematical models and MINOS is an solver for both linear and nonlinear programming problems [Brooke et al., 1992]. Solutions are applicable to obtain the suitable valve openings located at the rear of heat exchangers. If the adjustment of valve openings cannot satisfy the equivalent length, L_e, diameters of pipes located at the rear of heat exchangers should be changed.

RESULTS AND DISCUSSIONS

In this section, numerical examples are presented to illustrate the scope of this work. Example 1 illustrates the analysis of cooling water flows in a simple network. Further, Gauss-Seidel method and Newton-Raphson method are compared. Example 2 illustrates the usefulness of the manipulation algorithm in a complex heat exchangers network consisting of 10 heat exchangers.

1. Example 1 – Analysis of Cooling Water Flows

We used the algorithm developed in this study to analyze the cooling water flows. Calculated results of cooling water flow rates at each heat exchangers are shown in Fig. 6. Also the heads of each node were obtained in this example. Especially, head of node 1 (H_1=58.1 m) means that 5.7 atm of discharge pressure of pumps are at least required to operate this cooling water network. Convergences which are based on results are shown in Fig. 7. In this graph, the Gauss-Siedel method approached to convergence slowly compared to the Newton-Raphson method. For large complex networks, the Newton-Raphson method was found superior to solve the network problem. Fig. 5 shows a simple heat exchangers network which consists of 20 nodes (i=1,···,20) and 5 heat ex-

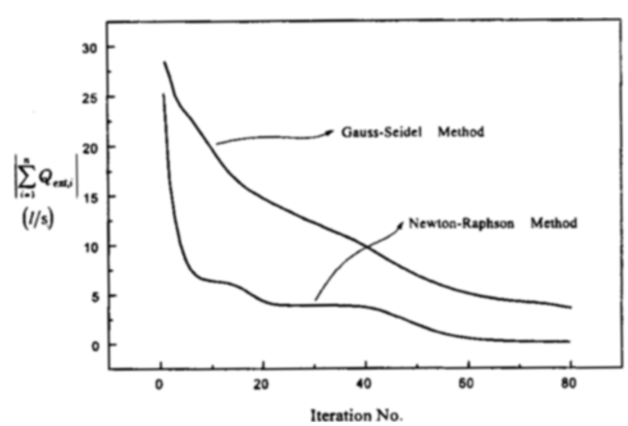

Fig. 7. Comparison of Gauss-Seidel method and Netwon-Raphson method.

Table 1. Type and data at each node for Example 1

Node	1	20	3-19
Type	QC	HC	–
External flows $Q_{ext,i}$ (*l*/s)	723.8	–	–
Head, H_i (m)	–	100	–

Table 2. Pipe diameters and lengths at the rear of heat exchangers in Example 1

NCES	Pipe		
	D (m)	L_{real} (m)	L_e (m)
7-19	0.254	72.2	74.0
9-18	0.254	56.2	58.0
11-17	0.254	57.2	59.0
13-16	0.254	56.2	58.0
15-16	0.305	64.2	66.0

changers named A, B, C, D, E, respectively. Types, external flows and heads at each node are shown in Table 1. Pipe diameters and lengths connected between each nodes are shown in Table 2. Lengths of pipes at the rear of heat exchangers are expressed as equivalent length including the valve equivalent lengths assuming all valves are 50% open for simplifications. Node types of QC mean the nodes in which the external flow rates are constant. Node types of HC mean the nodes in which the heads remain constant. Node 20 represents the cooling tower at atmosphere where head of the node is 100 m (head = node elevation + absolute pressure water in head). For this example, node 1 represents the cooling water supply pump in which constant flow rates of 723.8 *l*/s are entered into the node.

2. Example 2 – Manipulation of Cooling Water Flows

A heat exchangers network shown in Fig. 8 consists of 10 heat exchangers and 40 nodes and its pipe connections are intricate. In this problem, objective function is the differences between the present flow rates and the desired flow rates passing through the heat exchangers. The decision variables are equivalent pipe lengths located at the rear of the heat exchangers. They are bounded by the equivalent lengths within 10 m to 900 m and valve opening within 25% to 75% closed for operational reasons. If they are out of boundary, pipe diameters should be changed to adaptable size. Types and external flows at each node data and the data of pipe diameters and lengths connected between each nodes are shown in Table 3 and 4, respectively. The desired values of flow rates of each heat exchanger are shown in Fig. 9. The cooling water flow rates after adjustment are also shown in Fig. 9. In order to obtain these results, the degrees of valve openings and pipe diameter should be adjusted as shown in Table 5. The diameters of 3 pipes from among 10 pipes located at the rear of heat exchangers are changed to obtain the results.

Table 3. Type and data at each node for Example 2

Node	1	40	2-39
Type	QC	HC	–
External flows $Q_{ext,i}$ (*l*/s)	89.5	–	–
Head, H_i (m)	–	103	–

Table 4. Pipe diameters and lengths at the rear of heat exchangers in Example 2

NCES	Pipe		
	D (m)	L_{real}	L_e (m)
12-39	0.032	21.6	22.0
14-35	0.204	24.9	26.0
16-35	0.127	67.9	69.0
18-37	0.102	57.9	59.0
20-34	0.032	9.6	10.0
22-33	0.032	11.6	12.0
24-32	0.025	16.7	17.0
26-31	0.076	15.3	16.0
28-31	0.102	21.3	22.0
30-36	0.051	95.2	97.0

CONCLUSIONS

The method of pipe network analysis based on numerical techniques was applied to a heat exchangers network analysis effectively. This work presented an explicit algorithm for determining the flow rates of cooling water in a heat exchangers

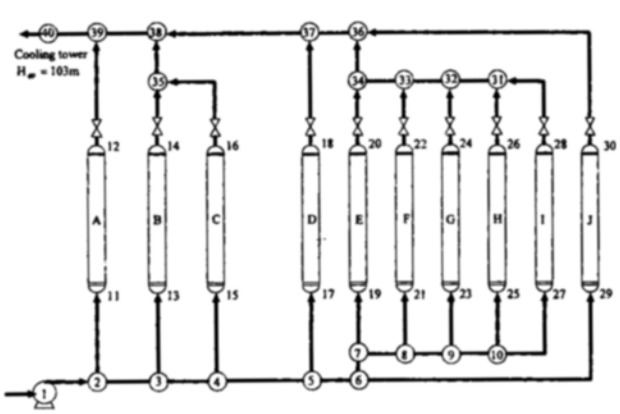

Fig. 8. A heat exchangers network for Example 2.

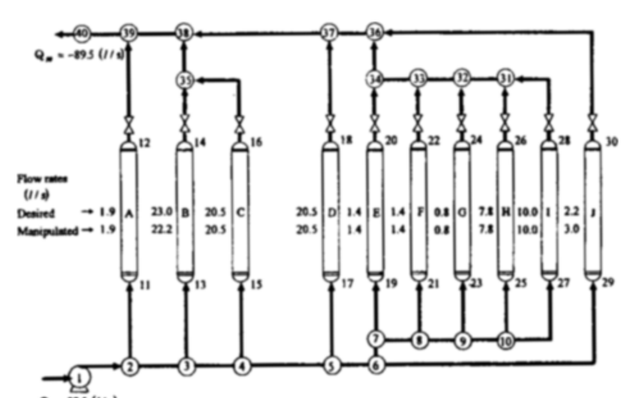

Fig. 9. Cooling water passing through heat exchangers for Example 2.

Table 5. Adjustment of valves and pipes located at the rear of heat exchangers for Example 2

NCES	Given		After adjustment	
	D (m)	Valve openings	D (m)	Valve openings
12-39	0.032	fully opened	–	30 % closed
14-35	0.204	〃	0.305	33 % 〃
16-35	0.127	〃	–	52 % 〃
18-37	0.102	〃	0.114	56 % 〃
20-34	0.032	〃	–	71 % 〃
22-33	0.032	〃	–	67 % 〃
24-32	0.025	〃	–	25 % 〃
26-31	0.076	〃	–	55 % 〃
28-31	0.102	〃	0.204	55 % 〃
30-36	0.051	〃	–	30 % 〃

network. Numerical example was presented to illustrate the facilities of the algorithm. Especially, the numerical method based on Newton-Raphson method excelled the others in aspects of convergency and accuracy. It also was found that the algorithm was effective to solve the large model which consists of 100 heat exchangers in the chemical plants.

The problem, which was concerned with differences of the present flow rates and the desired flow rates of the heat exchanger, was solved by adjustment of the equivalent lengths of pipes located at the rear of the heat exchangers. In this work, the optimization model was formulated in order to solve this problem in which equivalent lengths of pipes at the rear of heat exchangers were decision variables. According to the results obtained by GAMS/MINOS which is the nonlinear programming solver, adequate pipe diameters and satisfactory cooling water flow rates of each heat exchangers were obtained. This work will be useful to the operation of heat exchangers network using cooling water as well as the design of it.

ACKNOWLEDGEMENT

This work was supported by the Korea Science and Engineering Foundation (Grant No. 961-1110-061-2). This work also was supported in part by the Korea Science and Engineering Foundation through the Automation Research Center at Pohang University of Science and Technology.

NOMENCLATURE

C : conversion factor (4.727 for English units) [-]
 (10.47 for SI units) [-]
C_H : Hazen-Williams roughness coefficient (ex., 110 for new riveted steel pipe) [-]
D : pipe diameter [m]
H : water head (1 atm=10.33 m in water) [m]
HC : fixed head [-]
L : pipe length [m]
L_e : equivalent length [m]
L_{real} : real pipe length [m]
L_v : valve equivalent length [m]
Q : flow rates of cooling water [*l*/s]
Q^D : desired flow rates of cooling water [*l*/s]
QC : fixed flows rates [-]
NCES : node connected elements (ex., pipes...) [-]

Greek Letter

δ : degrees of valve openings [%]

Superscript

k : number of iteration

Subscripts

i : node
j : node connected to node i
ext : external demand or supply including its sign at the node.

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